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Global Optimal Design of Composite Laminates Including Failure Criteria Using Decomposition Techniques

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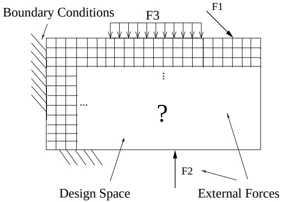
Problem Setting

- Discrete material optimization (DMO) with 0-1 design variables.



- Material selection among a discrete set of candidates materials.
- t:= Density of material block (design variables):

$$t_{ij} = \text{Density of the material } j \text{ used in the element } i.$$

- u_l:= Displacement due to the load condition f_l, $l = 1, \ldots, m$.
- M:= Available amount of material.

Problem Setting

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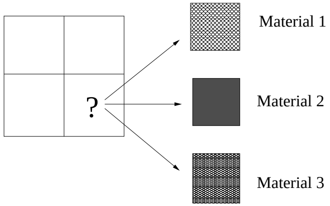

- Materials are defined by the specific stress-strain relationship.
- Material candidates could for example, be the same material oriented in specific angles (orthotropic materials).
- In this case, the problem becomes an angle selection problem.

Problem Formulation

- Minimum compliance, multi-material, local Failure problem:

$$
\begin{array}{llll}
\underset{t\in\mathbb{R}^n, u_l\in\mathbb{R}^d}{\text{minimize}} & \max\limits_{1\le l\le m}\{f_l^T u_l\} & & \text{(Compliance)} \\
s.t. & K(t)u_l = f_l, & l = 1,\dots,m & \text{(Equilibrium)} \\
\text{(P)} & \rho^T t \le M, & & \text{(Mass)} \\
 & t_{ij} \in \{0,1\}, & \forall\, i,j. & \text{(0-1 cond.)} \\
 & \sum_j t_{ij} = 1, & i = 1,...,n^c, & \text{(Mat. Select.)} \\
 & F(x,u_l) \le 0, & l = 1,\dots,m. & \text{(Local Failure.)}
\end{array}
$$

$$
K(t) = \sum_{i,j} t_{ij} K_{ij}, \quad K_{ij} \succeq 0, \qquad \text{linear elasticity.}
$$

Global Optimization

- Solve (P) to global optimality is a quite difficult task, even if no failure is considered.

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- Solve (P) to global optimality is a quite difficult task, even if no failure is considered.
- The task becomes even more difficult if the considered failure criterion does not have useful mathematical properties.
- Only few existing works in this area (and none for multimaterial problems).

Mixed Integer formulation: Features

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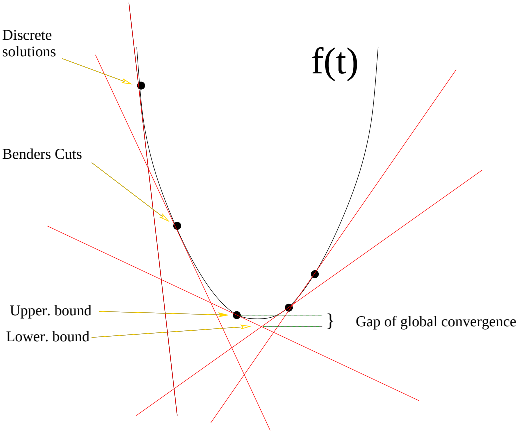
- The GBD method in general does not converge to global optima. Nevertheless, it does for the problem (P) when no failure criterion is considered.
- The main reason is the convexity of the compliance function as a function only in the design variable t.
- This issue must be taken into account when attacking the problem (P) (i.e., including local failure).

GBD Principle

Figure: Generalized Benders' Decomposition Method



GBD: Relaxed Master Problem (m=1)

$$\begin{aligned} \underset{t\in\{0,1\}^n, y\in\mathbb{R}}{\text{minimize}} \quad & y \\ s.t. \quad {\nu^k}^T t - y \le -2f^T u^k & \qquad k = 1, \ldots, N \\ \sum_{i,j} \rho_j t_{ij} \le M, & \\ \sum_j t_{ij} = 1 & \qquad \forall\, i \end{aligned} \tag{RMP}$$

- ${\nu^k}^T = -{u^k}^T \nabla_t K(t) u^k = -\left({u^k}^T K_1 u^k \;\; {u^k}^T K_2 u^k \;\ldots\; {u^k}^T K_n u^k\right)$
- u^k is the displacement solution to $K(t^k)u^k = f$.
- The t^k's is the solution of the $(k-1)$-th Relaxed Master Problem (RMP)

GBD Performance

- Heuristics to find candidate solutions improve the performance of the method.
- Solve Sub-MIP problem by GBD.
- Designs without failure criterion:
- C. Hvejsel, today 18h00, Solution with less than 2 % global optimality gap, for a multilayered problem of 23.000 design variables.

Local Failure Criteria

- Local failure criteria + Global Optimization: Possible?
- General types of failure criteria for composite structures
- Max stress, max strain, Tsai-Hill, Tsai-Wu, Puck, Cuntze, Hashin, etc
- It is not clear which is the most convenient failure criteria for composite structures.
- In general, all local failure criteria functions are not convex.
- GBD does not guarantee global solutions in this case.

Local Failure Criteria

- Max strain: $Au \leq b$
- Tsai- Wu/Tsai-Hill:

$$\frac{1}{2}u^T W_j(t)u + w_j(t)u \leq Y_j \qquad \forall\, j = 1, ..., n$$

$$W_j(t) = \sum_i t_{ij} W_{ij}, \quad w_j(t) = \sum_i t_{ij} w_{ij}$$

- $W(t)$ positive semidefinite matrix.
- If the failure is non convex a convex reformulation (if possible) is necessary for using GBD or any Global Optimization technique.

Local Failure Criteria

- ''Big-M'' reformulation of bilinear equations (Stolpe and Svanberg 2001)

$$z_{ij} = t_{ij} u$$

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-

$$\Longleftrightarrow$$

-

$$\begin{array}{rcl} t_{ij} c_{ij}^{min} \leq & z_{ij} & \leq t_{ij} c_{ij}^{max} \\ (1 - t_{ij}) c_{ij}^{min} \leq & u - z_{ij} & \leq (1 - t_{ij}) c_{ij}^{max} \end{array}$$

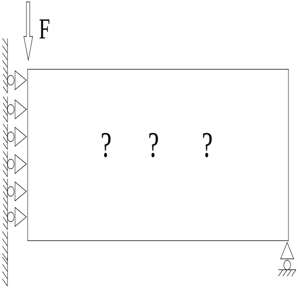
- c^{min}, c^{max} are convenient bounds.

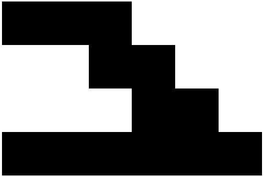
GBD for Problem (P)

- The ''Big-M'' reformulation can be applied to create GBD cuts for the max strain, max stress, Tsai-Hill/Wu failure criteria.
- However, the constraints generated by this method are in general too weak (there is no impact in performance).
- Algorithm becomes slower instead of becoming faster.
- Only possible for a few local failure criteria (if a convex reformulation exists).
- Even a feasible design is very difficult to find.

Toy Example:Topology Design, 24 DV

- No failure criterion, minimum compliance, connectivity imposed, optimal solution found





- Optimality gap: 0.5 %, CPU-time: 38[s], 54 iterations.
- maximum strain value: 5.397

Local Failure Criteria

- Strategy: Use this value as reference for setting the limit for the strain.

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- Attack (P) for a max strain failure criterion with the proposed GBD algorithm.
- Result: Algorithm runs for 12[h], and not a single feasible solution is found.
- Conclusion: Failure feasibility constraints from the GBD method are too weak for expecting an acceptable performance.

Local Failure Criteria

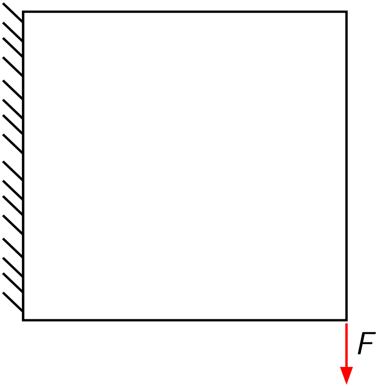
- Alternative: Solve the problem (P) in two stages. First, without the local failure criterion.
- When the problem is solved to optimality, reset the Upper bound to $+\infty$. Restart the GBD algorithm considering the local failure criterion in the problem formulation.
- If the current design at iteration k, t^k, is infeasible for the local failure, include a single linear constraint preventing t^k (and only t^k) to be feasible in the master problem.

$$c^T t^k \leq b^k, \quad c \in \mathbb{R}^{n+1}, b^k \in \mathbb{R}.$$

- The method can be used for any kind of failure function $F(x, u_l)$ (no special properties are needed).

Local Failure Criteria

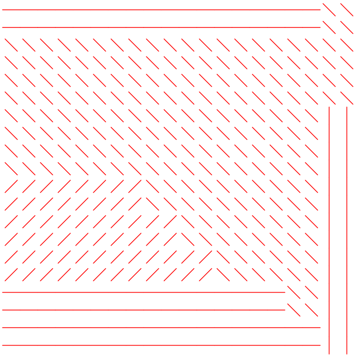
Figure: Example



- Angle selection problem: +45, -45, 0, 90.
- 400 FE discretization.
- 100 design element discretization x 4 candidate angles: 400 DV.

No Failure Solution

- No failure criterion, minimum compliance, solution found.



- Optimality gap: 2.64 %, compliance: 15.4716.
- maximum strain value: 5.74397E-03. CPU-time: 12[h]

Local Failure Criteria

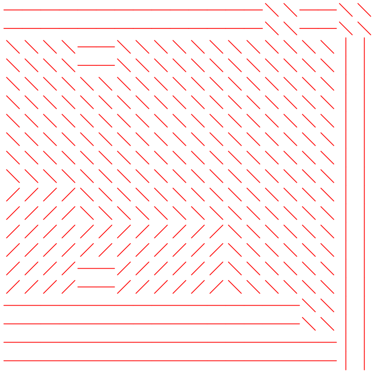
- Strategy: Use this value as reference for setting the limit for the strain.
- Strain limit imposed

$$\| \epsilon \| \leq 5.74397 \cdot 10^{-3} - 1 \cdot 10^{-8} = 5.74396 \cdot 10^{-3}.$$

- Attack (P) with the proposed GBD algorithm.

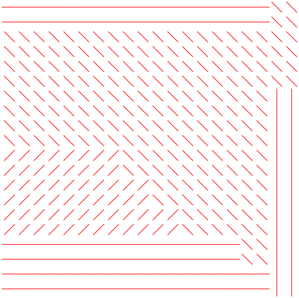
Local Failure Criteria

- Failure criterion included, solution found.

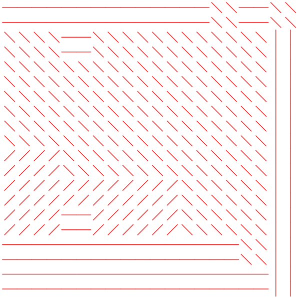


- Optimality gap: 2.87 %, Objective Value: 15.4902.
- maximum strain value: $5.74375 \cdot 10^{-3}$. CPU-Time: 43[h]

Comparison



(a) No local failure



(b) Max strain failure criterion

Summary and Conclusions

- Generalized Benders' Decomposition can solve medium size structural design problems to optimality.
- The inclusion of local failure criteria makes the feasible set smaller.
- $\Longrightarrow$ Faster convergence is expected. However, the opposite occurs.

Summary and Conclusions

- The minimum compliance problem problem (P) + local failure.

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Summary and Conclusions

- The minimum compliance problem problem (P) + local failure.

  can be attacked by Generalized Benders' Decomposition.
- $\Longrightarrow$ The algorithm converges theoretically to a global minimum

  solution in a finite number of steps, or stops with an infeasibility flag (if the problem is infeasible).
- The method can be used for almost any failure criterion, independently of convexity assumptions.

Thanks for your attention!